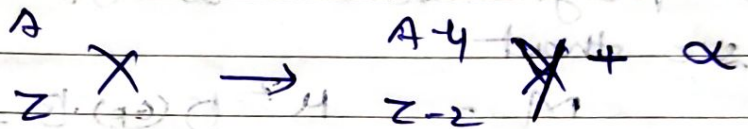


Question Q. → what are the alpha and Beta decay?

Answer → In any Nuclei Emission of an Alpha particle is a combination of 2 protons and 2 neutrons, so the  $Z$  of parent nuclei reduced by 2 unit and mass number reduced by 4 unit.



above eq<sup>n</sup> shows the  $\alpha$  decay but here we will see why  $\alpha$ -decay occurs.

Alpha emission is a coulomb repulsion effect. It becomes increasingly important for heavy nuclei because the disruptive Coulomb force increases with size at a faster rate (namely, as  $Z^2$ ) than does the specific nuclear binding force, which increases approximately as  $A$ . Why  $\alpha$  particle chosen as the agent for the spontaneous carrying away

of positive charge? When we call a process spontaneous we mean that some kinetic energy has suddenly appeared in the system for no apparent cause; this energy must come from a decrease in the mass of the system. The  $\alpha$  particle, because it is very stable and tightly bound structure, has a relatively small mass compared with the mass of its separate constituents. It is particularly favored as an emitted particle if we hope to have the disintegration products as light as possible and thus get the largest possible release of kinetic energy.

### Basic $\alpha$ Decay Process:

Let's first consider the conservation of energy in the  $\alpha$  decay process. We assume the initial decaying nucleus  $X$  to be at rest. Then the energy of the initial system is just the rest energy of  $X$ ,  $m_X c^2$ . The final state <sup>consist</sup> of  $X'$  and  $\alpha$ , each of which will be in motion (to conserve linear momentum). Thus the final total energy is  $m_X c^2 + T_{X'} + m_\alpha c^2 + T_\alpha$ , where  $T$  represents the kinetic energy of the final particles. Thus conservation of energy gives

$$m_x c^2 = m_{x'} c^2 + T_{x'} + m_\alpha c^2 + T_\alpha \quad \text{--- (1)}$$

$$(m_x - m_{x'} - m_\alpha) c^2 = T_{x'} + T_\alpha \quad \text{--- (2)}$$

The quantity on the left side of eqn (2) is the net energy released in the decay called the  $Q$  value:

$$Q = (m_x - m_{x'} - m_\alpha) c^2 \quad \text{--- (3)}$$

and decay will occur spontaneously only if  $Q > 0$ .

Beta decay:-

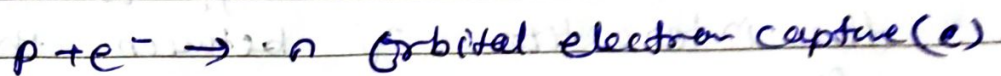
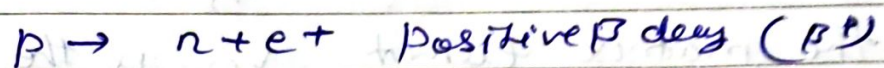
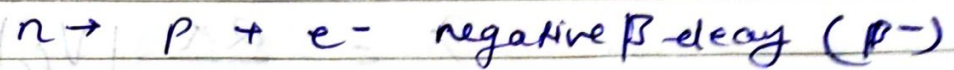
The most basic  $\beta$  decay process is the conversion of a proton to neutron or a neutron into a proton. In  $\beta$  decay,  $Z$  and  $N$  change both by one unit.

$$Z \rightarrow Z \pm 1, \quad N \rightarrow N \pm 1, \quad \text{so that} \\ A = Z + N \text{ remains constant!}$$

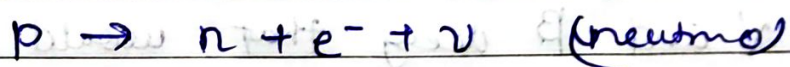
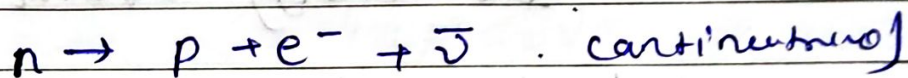
Thus  $\beta$  decay provides a convenient way for an unstable nucleus to reach a stable structure.

In contrast with  $\alpha$  decay, progress in understanding  $\beta$ -decay has been achieved at an extremely slow pace,

often the experimental results have created new puzzles that challenged existing theories. New theory by  $\beta$ -decay process are thus



These processes are not complete, for there is yet another ~~set~~ particle (a neutrino or anti-neutrino) involved in each. The latter only take place occur only for protons bound in nuclei. They are energetically forbidden for free protons or free protons in hydrogen atom.



Question Q State and prove Fermi theory of  $\beta$ -decay.

Answer  $\rightarrow$  Enrico Fermi developed a theory for allowed  $\beta$ -decay, based on Pauli's neutrino hypothesis. Fermi used the result of direct-time dependent

perturbation theory according to which the rate of transition from an initial state  $i$  to final state  $f$ . (Golden Rule)

$$\lambda = \frac{2\pi}{\hbar} |V_{fi}|^2 \rho(E_f) \quad (1)$$

the matrix element  $V_{fi}$  is the integral of the interaction  $V$  between the initial and final quasi-stationary states of the system:

$$V_{fi} = \int \psi_f^* V \psi_i \cdot dV \quad (2)$$

The factor  $\rho(E_f)$  is the density of final states, which can also be written as  $dn/dE_f$ , the number  $dn$  of final states in the energy interval  $dE_f$ .

Fermi did not know the mathematical form of  $V$  and  $\rho$  decay that would have permitted calculations using Eq<sup>n</sup> (1) & (2). Instead he considered all possible forms consistent with ~~the~~ special relativity, and he showed that  $V$  could be replaced with one of the five mathematical operators  $O_\lambda$ , where the subscript  $\lambda$  gives the form of the operator  $O$ :  
 $\lambda = V$  (vector),  $A$  (axial vector),

S (scalar), P (pseudoscalar) or T (tensor). Which of these is correct for  $\beta$  decay can be revealed only through experiments + study the symmetries and the spatial properties of the decay products.

The final state wavefunction, must include not only nuclei but also the electron and neutrino

$$\langle f_i | = \int [\psi_f^* \psi_e^* \psi_\nu^*] O_i \psi_i dV \quad (4)$$

If the electron is confined to a box of volume  $V$ , then the number of final electron states  $dn_e$ , corresponding to momentum in range  $p$  to  $p + dp$ , is

$$dn_e = \frac{4\pi p^2 dp V}{h^3} \quad (5)$$

similarly for neutrino

$$dn_\nu = \frac{4\pi q^2 dq V}{h^3} \quad (6)$$

$$d^2n = dn_e dn_\nu = \frac{(4\pi)^2 V^2 q^2 dq p^2 dp}{h^6} \quad (7)$$

The electron wave function for non-relativistic

$$\psi_e(\mathbf{r}) = \frac{1}{\sqrt{V}} e^{i\mathbf{p}\cdot\mathbf{r}/\hbar} \quad (6)$$

$$\psi_e(\mathbf{r}) = \frac{1}{\sqrt{V}} e^{i\mathbf{q}\cdot\mathbf{r}/\hbar}$$

we will get

$$d\lambda = \frac{2\pi}{\hbar} g^2 |M_{fi}|^2 (2\pi)^2 \frac{p^2 dp q^2 dq}{h^6} \frac{d\Omega}{dE_f} \quad (7)$$

$$N \propto p^2 (q - T_e)^2 F(Z', p) |M_{fi}|^2 S(p, q) \dots$$