Question (4) → what are the alpha and Beta decay?

Answer → In any Nuclei Emission of an Alpha particle is a combination of 2 proton and 2 neutron. so the Z of parent nuclei reduced by 2 unit and mass number reduced by 4 unit.

$$ {}^{A}_{Z}X \rightarrow {}^{A-4}_{Z-2}Y + \alpha $$

above eqn shows the α decay but now we will see why α-decay occurs.

Alpha emission is a coulomb repulsion effect. It becomes increasingly important for heavy nuclei because the disruptive Coulomb force increases with size at a faster rate (namely as Z^2) than does the specific nuclear binding force, which increases approximately as A. why α particle choosen as the agent for the spontaneous carrying away of positive charge? When we call a process spontaneous we mean that some kinetic energy has suddenly appeared in the system for no apparent cause; this energy must come from a decrease in the mass of the system. The α particle, because it is very stable and tightly bound structure, has a relatively small mass compared with the mass of its separate constituents. It is particularly favored as an emitted particle if we hope to have the disintegration products as light as possible and thus get the largest possible release of kinetic energy.

Basic α-Decay Process:-

Lets first consider the conservation of energy in the α decay process. We assume the initial decaying nucleus X to be at rest. Then the energy of the initial system is just the rest energy of X, $m_X c^2$. The final state consist of X' and α, each of which will be in motion (to conserve linear momentum). Thus the final total energy is $m_{X'} c^2 + T_{X'} + m_\alpha c^2 + T_\alpha$, where T represents the kinetic energy of the final particles. Thus conservation of energy gives

$$m_X c^2 = m_{X'} c^2 + T_{X'} + m_\alpha c^2 + T_\alpha \quad \text{--- (1)}$$

$$(m_X - m_{X'} - m_\alpha)c^2 = T_{X'} + T_\alpha \quad \text{--- (2)}$$

The quantity on the left side of eqn (2) is the net energy released in the decay called the Q value:

$$Q = (m_X - m_{X'} - m_\alpha)c^2 \quad \text{--- (3)}$$

and decay will occur spontaneously only if $Q > 0$.

Beta decay:- The most basic β decay process is the conversion of a proton to neutron or a neutron into a proton. In a nucleus, β decay changes both Z and N by one unit

$$Z \to Z \pm 1, \quad N \to N \pm 1, \quad \text{so that}$$
$$A = Z + N \text{ remains constant!}$$

Thus β decay provides an convenient way for an unstable nucleus to stable structure.

In contrast with α decay, progress in understanding β-decay has been achieved at an extremely slow pace, often the experimental results have created new puzzles that challanged existing theories. New theory by β-decay process are then

$$n \rightarrow p + e^-$$ negative β-decay (β^-)

$$p \rightarrow n + e^+$$ Positive β decay (β^+)

$$p + e^- \rightarrow n$$ Orbital electron capture (e)

These processes aren not complete, for there is yet another ~~ant~~ particle (a neutrino or anti-neutrino) involved in each. The later only two processes occur only for protons bound in nuclei. They are energetically forbidden for free protons or free protons in hydrogen atom.

$$n \rightarrow p + e^- + \bar{\nu}$$ (antineutrino)

$$p \rightarrow n + e^- + \nu$$ (neutrino).

Question → ~~State~~ and Prove Fermi theory of β-decay.

Answer → Enrico Fermi developed a theory ~~of~~ for allowed of β-decay based on pauli's neutrino hypothesis. Fermi used the result of Direact - time dependent perturbation theory according to which the rate of transition from an initial state i to final state f. (Golden Rule)

$$\lambda = \frac{2\pi}{\hbar} |V_{fi}|^2 \rho(E_f) \quad \text{---(1)}$$

the matrix element V_{fi} is the integral of the interaction V between the initial and final quasi-stationary states of the system:

$$V_{fi} = \int \psi_f^* V \psi_i \, dv \quad \text{---(2)}$$

The factor $\rho(E_f)$ is the density of final states, which can also be written as dn/dE_f, the number dn of final states in the energy interval dE_f.

Fermi did not know the mathematical form of V and β decay that would have permitted calculations using Eq^n (1) & (2). Instead he considered all possible forms consistent with ~~the~~ special relativity, and he showed that V could be replaced with one of the five mathematical operators O_x, where the subscript x gives the form of the operator O:

$X = V$ (vector), A (axial vector),

S (scalar), P (pseudoscalar) or T (Tensor). W
of these is correct for β decay can
revealed only through experiments t
study the symmetries and the spati
properties of the decay products.

The final state wavefunction must includ
not only nuclei but also the electron an
neutrino

$$V_{fi} = g \int [\psi_f^* \varphi_e^* \varphi_\nu^*] O_x \psi_i \, dv \quad \text{—(}$$

If the electron is confined to a box of volume V, then the number of final electron states dn_e, corresponding to mom in range p to dp + p, is

$$dn_e = \frac{4\pi p^2 dp V}{h^3} \quad \text{—— (4)}$$

similarly for neutrino

$$dn_\nu = \frac{4\pi q^2 dq V}{h^3} \quad \text{—— (5)}$$

$$d^2n = dn_e dn_\nu = \frac{(4\pi)^2 V^2 q^2 dq \, p^2 dp}{h^6} \quad \text{—— (6)}$$

The electrons and neutrino have wavefunctions

$$\psi_e(r) = \frac{1}{\sqrt{V}} e^{ip.r/\hbar} \quad \text{---} (6)$$

$$\psi_\nu(r) = \frac{1}{\sqrt{V}} e^{iq.r/\hbar}$$

we will get

$$d\lambda = \frac{2\pi}{\hbar} g^2 |M_{fi}|^2 (4\pi)^2 \frac{p^2 dp\, q^2 dq}{h^6 \, dE_f} \text{ --- } (7)$$

$$N(p) \propto p^2 (Q - T_e)^2 F(Z', p) |M_{fi}|^2 S(p,q) \ldots$$

process.